BLIND IDENTIFICATION OF UNDERDETERMINED MIXTURES BASED ON THE HEXACOVARIANCE AND HIGHER-ORDER CYCLOSTATIONARITY

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ABSTRACT

In this work, we consider the problem of blind identification of underdetermined mixtures in a cyclostationary context relying on sixth-order statistics. We propose to exploit the cyclostationarity at higher orders by taking into account the knowledge of source cyclic frequencies in the sample estimator of the observation hexacovariance. Two blind identification algorithms based on the proposed estimator are considered and their performances are tested by means of computer simulations. Our simulation results show that significant improvements can be obtained when both second and fourth-order cyclo-stationarities are exploited.

***Index Terms*—** Blind identification, underdetermined mixtures, cyclostationarity, hexacovariance. [1]

1. INTRODUCTION

Blind identification and blind source separation methods have been successfully applied in multidisciplinary contexts, including radiocommunications, sonar, radar, biomedical signal processing and data analysis, just to mention a few. A widespread class of these methods rely on independent component analysis by means of higher-order statistics [1]. In this context, a problem that have attracted a particular interest is that of blind identification of underdetermined mixtures (i.e. when we have more sources than sensors). Several solutions have been proposed in the literature to solve this problem (see, e.g. [2, 3, 4, 5, 6, 7] and references therein). The solution proposed in these works may resort to second, fourth or sixth-order statistics of the output data.

In several applications such as radiocommunications and passive listening, the sources may be nonstationary, and they are often (quasi)-cyclostationary. This property appears as soon as the observations are oversampled and/or when the different sources have different bandwidth. The work [8] addressed the behavior of second- and four-order blind source separation algorithms in a cyclostationary context. The authors proposed to exploit the cyclostationary property of the sources by adding a correction term to the standard sample estimator of the quadricovariance which takes into account the known cyclic frequencies of the received sources. The results presented in [8] show that the performance of fourth-order statistics based blind source separation algorithms can be considerably improved when the proposed estimator is used. The cyclostationarity property has no yet been considered for cumulant estimators of orders higher than four. The works [9, 10] exploit sixth-order statistics in the blind identification problem. Following these works, a family of blind identification algorithms called BIOME (Blind Identification of Overcomplete MixturEs) was proposed in [4]. Although powerful, these algorithms do not take into account the cyclostationary nature of the sources since they rely on the standard sample estimate of the hexacovariance.

In this work, we propose to exploit the higher-order cyclostationarity in the blind identification problem. Motivated by the results of [8, 11] on one hand, and of [4] on the other hand, we propose to take into account the known cyclic frequencies of the sources in the calculation of the empirical estimator of the hexacovariance. Two blind identification algorithms based on the proposed estimator are tested. The first one is the 6-BIOME algorithm of [4], also referred to as "BIRTH" in the 6th order case, while the second one is a direct minimization of a tensor model fitting error by an iterative algorithm.

2. PROBLEM DEFINITION AND ASSUMPTIONS

Consider a noisy mixture of P statistically independent narrowband sources received by an array of M sensors. The vector $\mathbf{y}(k)$ containing the discrete-time version of the complex envelopes of the received signal at the sensor outputs can be modeled according to the following classical linear model:

$$\mathbf{y}(k) = \sum_{p=1}^{P} s_p(k)\mathbf{h}_p + \mathbf{n}(k) \doteq \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \qquad (1)$$

where $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_P] \in \mathbb{C}^{M\times P}$, $\mathbf{s}(k) = [s_1(k), \ldots, s_P(k)]^T \in \mathbb{C}^P$ and $\mathbf{n}(k) \in \mathbb{C}^M$ are the mixing matrix, source and noise random vectors, respectively. It is assumed that for any fixed time index k, $\mathbf{s}(k)$ and $\mathbf{n}(k)$ are statistically independent. We are interested in the case where the received source signals are *cyclostationary* and have a *nonzero carrier residue*. These properties are generally verified in interception or passive listening applications. In this scenario, the input-output model (1) may be too restrictive so that we adopt following observation model:

$$\begin{aligned} \mathbf{y}(k) &= \sum_{p=1}^{P} s_p(k) e^{-\jmath(2\pi\Delta f_p k T_s + \phi_p)}\mathbf{h}_p + \mathbf{n}(k) \\ &= \mathbf{H}\bar{\mathbf{s}}(k) + \mathbf{n}(k), \end{aligned} \qquad (2)$$

where $\bar{\mathbf{s}}(k) = [\bar{s}_1(k), \ldots, \bar{s}_P(k)]^T$ is the new source signal vector, with $\bar{s}_p(k) \doteq s_p(k)e^{-\jmath(2\pi\Delta f_p kT_s+\phi_p)}$, while Δf_p and ϕ_p are, respectively, the carrier residue and phase of the p-th source and T_s is the sampling period. The sources are assumed to have the same symbol period T, while the observations are sampled at the Nyquist rate, i.e. $T_s \geq T/2$.

Before to proceed, the following hypotheses are assumed:

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H1. The sources $s_1(k), \ldots, s_P(k)$ are non-Gaussian, cyclostationary, cycloergodic, and mutually uncorrelated up to order 6;

H2. The noise vector $\mathbf{n}(k)$ is stationary and ergodic, following a complex-valued Gaussian distribution;

H3. The sixth order marginal source cumulants are not null and have all the same sign;

H4. The mixing matrix $\mathbf{H}$ does not contain collinear columns.

The problem is to identify the mixing matrix $\mathbf{H}$ (up to trivial column permutation and scaling) and, possibly, the source vector $\mathbf{s}(k)$ from the only knowledge of the observation vector $\mathbf{y}(k)$ by means of an estimation of its associated sixth-order statistics. Recall that we are interested in the so-called underdetermined case, which means that we have $P > M$.

After the identification of the mixing matrix, we consider a Maximum-A-Posteriori (MAP) criterion to estimate the source vector $\mathbf{s}(k)$ by means of an exhaustive search over the known finite alphabet of the sources. This methodology will be used for evaluating the Bit-Error-Rate (BER) of the proposed blind identification algorithms exploiting higher-order cyclostationarity.

3. HIGHER-ORDER STATISTICS IN THE PRESENCE OF CYCLOSTATIONARITY

As discussed in [8, 11], in the cyclostationary context the covariance function of $\overline{s}_p(k)$ admits a Fourier series expansion over the set $\Gamma_p = \{\alpha_p\}$ of *cyclic frequencies*, where the coefficients of this expansion correspond to the *cyclic covariances* of the p-th source. Therefore, the covariance matrix of $\mathbf{y}(k)$ contains cyclic frequencies of all the sources belonging to the set $\{\Gamma_1, \ldots, \Gamma_P\}$. In the general case, the N-th order cyclic covariance of $\mathbf{y}(k)$ associated with a given cyclic frequency α is defined by:

$$c_{i_1,\ldots,i_p,\mathbf{y}}^{i_{p+1},\ldots,i_N}(\alpha) = \left\langle \overline{c}_{i_1,\ldots,i_p,\mathbf{y}}^{i_{p+1},\ldots,i_N}(k) e^{-\jmath 2\pi\alpha k T_s} \right\rangle_d \tag{3}$$

where $\overline{c}_{i_1,\ldots,i_p,\mathbf{y}}^{i_{p+1},\ldots,i_N}(k)$ are the N-th order output cumulants and $\left\langle f(k) \right\rangle_d \doteq \lim_{K\to\infty}(1/K)\sum_{k=1}^{K} f(k)$ corresponds to the discrete-time temporal mean operation of $f(k)$ over an infinite number of samples. We assume that the set $\{\Gamma_1, \ldots, \Gamma_P\}$ of cyclic frequencies of the sources is known. It is worth noting that a set of these cyclic frequencies depends on the nonzero carrier residues of the sources $\Delta f_1, \ldots, \Delta f_P$.

The main motivation for considering the cyclostationarity property in the blind identification and source separation problems comes from the fact that, when the observations are (quasi)-cyclostationary, the time-averaged cumulants $\widehat{c}_{i_1,\ldots,i_p,\mathbf{y}}^{i_{p+1},\ldots,i_N}$ (i.e. those calculated at the zero cyclic frequency only) generate, as K becomes infinite, an "apparent" (biased) estimation of the cumulants instead of the true cumulants. This is a consequence of the time dependence of the statistics of the data for (quasi)-cyclostationary sources [8]. This estimation bias can, however, be compensated by adding a correction term to the sample estimator that takes into account all the nonzero cyclic frequencies present in the observed data. We propose to exploit the cyclostationarity property by means of sixth-order statistics (hexacovariance). It is worth mentioning that the hexacovariance has been considered in [10, 4] for blind identification of the underdetermined mixtures. However, these works have not exploited the cyclostationarity property of the sources. Here, we rely on the results of [8, 11] (which have considered the quadricovariance) to address the performance of two blind identification algorithms based on the hexacovariance. These algorithms are presented in Section 4. Our goal is to evaluate the potential improvements obtained when higher-order cyclic moments are exploited.

Proposed hexacovariance estimator: Let $\overline{c}_{i_1,i_2,i_3,\mathbf{y}}^{i_4,i_5,i_6}$ be an element of the hexacovariance of the observations $\mathbf{y}(k)$. The cyclic estimator $\widehat{\overline{c}}_{i_1,i_2,i_3,\mathbf{y}}^{i_4,i_5,i_6}$ of the hexacovariance based on K snapshots of the data is given by:

$$\begin{aligned}
\widehat{\overline{c}}_{i_1,i_2,i_3,\mathbf{y}}^{i_4,i_5,i_6} \doteq\ & \widehat{r}_{i_1,i_2,i_3}^{i_4,i_5,i_6}(0) \\
& - [3]\Big(\sum_{\substack{\alpha,\beta \\ \alpha+\beta=0}} \widehat{r}_{i_1,i_2,i_3}^{i_4}(\alpha)\widehat{r}_{i_5}^{i_6}(\beta) \Big) \\
& - [9]\Big(\sum_{\substack{\alpha,\beta \\ \alpha+\beta=0}} \widehat{r}_{i_1,i_2}^{i_4,i_5}(\alpha)\widehat{r}_{i_3}^{i_6}(\beta) \Big) \\
& - [3]\Big(\sum_{\substack{\alpha,\beta \\ \alpha+\beta=0}} \widehat{r}_{i_1,i_2}(\alpha)\widehat{r}_{i_3}^{i_4,i_5,i_6}(\beta) \Big) \\
& + 2[9]\Big(\sum_{\substack{\alpha,\beta,\gamma \\ \alpha+\beta+\gamma=0}} \widehat{r}_{i_1,i_2}(\alpha)\widehat{r}_{i_3}^{i_4}(\beta)\widehat{r}^{i_5,i_6}(\gamma) \Big) \\
& + 2[6]\Big(\sum_{\substack{\alpha,\beta,\gamma \\ \alpha+\beta+\gamma=0}} \widehat{r}_{i_1}^{i_4}(\alpha)\widehat{r}_{i_2}^{i_5}(\beta)\widehat{r}_{i_3}^{i_6}(\gamma) \Big)
\end{aligned} \tag{4}$$

where

$$\widehat{r}_{i_1,i_2,i_3}^{i_4,i_5,i_6}(0) \doteq \frac{1}{K}\sum_{k=1}^{K} y_{i_1}(k)y_{i_2}(k)y_{i_3}(k)y_{i_4}^*(k)y_{i_5}^*(k)y_{i_6}^*(k)$$

is the estimated sixth-order moment at the zero frequency and

$$\begin{aligned}
& \widehat{r}_{i_1,\ldots,i_p}^{i_{p+1},\ldots,i_{p+q}}(\alpha) \\
& \doteq \frac{1}{K}\sum_{k=1}^{K} y_{i_1}(k)\cdots y_{i_p}(k)y_{i_{p+1}}^*(k)\cdots y_{i_{p+q}}^*(k)e^{-\jmath 2\pi\alpha k T_s}.
\end{aligned}$$

is the estimated $(p+q)$-th order moment evaluated at the cyclic frequency α, where $p+q \leq 6$. In the compact expression (4), $[n]$ denotes the McCullagh bracket notation representing the existence of n monomials of the same order that arises by permuting separately either superscripts or subscripts [12] [13]. Finally, note that the values taken by α, β, and γ satisfying $\alpha+\beta+\gamma=0$, are those of the known cyclic frequencies.

Remark: The proposed estimator of the hexacovariance is only "approximately unbiased" since we have ignored the bias introduced by the estimated sixth-order moment. This approximation has two motivations. First, the complexity associated with the calculation of the unbiased sixth-order moment estimator is prohibitive. Second, the performances obtained with the proposed approximation are satisfactory, as it will be shown in Section 5.

4. BLIND IDENTIFICATION ALGORITHMS

In this section, we exploit the higher-order cyclostationarity of the sources by considering two blind identification algorithms capable of identifying underdetermined mixtures. The first one is the sixth-order version of the BIOME family of algorithms and is called 6-BIOME [4]. The second algorithm is based on the same modeling, but the 6th order cumulants are stored in a 3rd order tensor instead of a matrix. Before presenting these algorithms, we introduce some notation and properties associated with the matrix representations of the hexacovariance tensor. Thanks to the multilinearity proprety of cumulants, the hexacovariance $\overline{c}_{i_1,i_2,i_3,\mathbf{y}}^{i_4,i_5,i_6}$ of the observations $\mathbf{y}(k) = \mathbf{H}\overline{\mathbf{s}}(k)$ is a sixth-order rank-P tensor $\mathcal{C} \in \mathbb{C}^{M\times M\times M\times M\times M\times M}$ which can be written as

$$\mathcal{C}_{\mathbf{y}} = \sum_{p=1}^{P} \kappa_p (\mathbf{h}_p \circledcirc \mathbf{h}_p \circledcirc \mathbf{h}_p \circledcirc \mathbf{h}_p^* \circledcirc \mathbf{h}_p^* \circledcirc \mathbf{h}_p^*), \tag{5}$$

where $\circledcirc$ denotes the outer product between vectors and κ_p is the marginal 6th order cumulant of the p-th source. The latter model is sometimes referred to as "PARAFAC".

Symmetric matrix factorization

The overall information contained in the hexacovariance tensor $\mathcal{C}_{\mathbf{y}}$ defined in (5) can be organized in a symmetric matrix $\mathbf{C}_{\mathbf{y}} \in \mathbb{C}^{M^3\times M^3}$ defined as follows [4]:

$$\begin{aligned} \mathbf{C}_{\mathbf{y}} &= \sum_{p=1}^{P} \kappa_p \big(\mathbf{h}_p \otimes \mathbf{h}_p \otimes \mathbf{h}_p^*\big)\big(\mathbf{h}_p \otimes \mathbf{h}_p \otimes \mathbf{h}_p^*\big) \\ &= \mathbf{H}^{\odot 3}\boldsymbol{\Delta}_{\mathbf{s}}(\mathbf{H}^{\odot 3})^{\mathrm{H}}, \end{aligned} \tag{6}$$

where $\otimes$ and $\odot$ denote, respectively the Kronecker and Khatri-Rao products, $\mathbf{H}^{\odot 3} = \mathbf{H} \odot \mathbf{H} \odot \mathbf{H}^* \in \mathbb{C}^{M^3\times P}$ and $\boldsymbol{\Delta}_{\mathbf{s}} \in \mathbb{C}^{P\times P}$ is a diagonal matrix containing the marginal source cumulants $\kappa_1, \ldots, \kappa_P$ along the main diagonal. The 6-BIOME algorithm briefly, which is presented in Section 4.1, relies on model (6).

Non-symmetric matrix factorizations

We can organize the information contained in the hexacovariance tensor in alternative (non-symmetric) matrix forms. Let us consider the following one:

$$\mathbf{C}_{\mathbf{y}}^{'} = \Big(\mathbf{H} \odot \mathbf{H} \odot \mathbf{H} \odot \mathbf{H}^* \odot \mathbf{H}^*\Big)\mathbf{H}^{(3)\mathrm{T}}, \in \mathbb{C}^{M^5\times M} \tag{7}$$

or, alternatively,

$$\mathbf{C}_{\mathbf{y}}^{'} = \Big(\mathbf{H}^{(1)} \odot \mathbf{H}^{(2)}\Big)\mathbf{H}^{(3)\mathrm{T}}, \tag{8}$$

where $\mathbf{H}^{(1)} = \mathbf{H}\odot\mathbf{H}\odot\mathbf{H}$, $\quad \mathbf{H}^{(2)} = \mathbf{H}^*\odot\mathbf{H}^*$ and $\mathbf{H}^{(3)} = \mathbf{H}^*\boldsymbol{\Delta}_{\mathbf{s}}$. The factorization (8) is an equivalent third-order PARAFAC model of dimensions $M^3 \times M^2 \times M$ representing the hexacovariance tensor. In Section 4.2, this factorization is exploited for blind identification by means of the Levenberg-Marquardt (LM) algorithm.

4.1. Sixth-order BIOME (6-BIOME)

The 6-BIOME algorithm is reminiscent of the BIRTH (Blind Identification using Redundancies in the daTa Hexacovariance matrix) algorithm proposed in [9] and later improved in [10]. It exploits the multilinear algebraic structure of the hexacovariance by solving a joint approximate diagonalization problem based on the symmetrically unfolded matrix factorization (6) of the estimated hexacovariance (4). Following the idea of [4, 9], we can write the square-root of (6) as:

$$(\mathbf{C}_{\mathbf{y}})^{1/2} = \mathbf{H}^{\odot 3}\,\boldsymbol{\Delta}_{\mathbf{s}}\,\mathbf{V} \tag{9}$$

where $\mathbf{V}$ is a unitary matrix. The 6-BIOME algorithm is summarized as follows:

6-BIOME ALGORITHM

1. Estimate the hexacovariance tensor using all the cyclic moments (4) and form $\mathbf{C}_{\mathbf{y}}$;
2. Compute the square-root $(\mathbf{C}_{\mathbf{y}})^{1/2} \in \mathbb{C}^{M^3\times P}$ from the Eigen-Value Decomposition (EVD) of $\mathbf{C}_{\mathbf{y}}$;
3. Slice $(\mathbf{C}_{\mathbf{y}})^{1/2}$ into M matrix blocks $\boldsymbol{\Gamma}_m \in \mathbb{C}^{M^2\times P}$;
4. Find $\mathbf{V}$ by solving a simultaneous diagonalization problem from $M(M-1)/2$ Hermitian matrices $\boldsymbol{\Theta}_{m,n} \doteq \boldsymbol{\Gamma}_m^{\dagger}\boldsymbol{\Gamma}_n$;
5. Calculate $\widehat{\mathbf{H}}^{\odot 3} = (\mathbf{C}^{\mathbf{y}})^{1/2}\mathbf{V}^{\mathrm{H}}$;
6. Arrange each of the P columns of $\widehat{\mathbf{H}}^{\odot 3}$ in vector $\mathbf{b}_m \in \mathbb{C}^{M^3}$;
7. Transform each vector $\mathbf{b}_m \in \mathbb{C}^{M^3}$ in a set of M matrices $\mathbf{B} \in \mathbb{C}^{M\times M}$ and calculate the dominant eigenvector $\mathbf{h}_m$ of each of these matrices;
8. Form $\widehat{\mathbf{H}}$, the columns of which are the vectors $\mathbf{h}_m$.

4.2. Identification with Levenberg-Marquardt (PARAFAC-LM)

The second algorithm is based on the 3rd order tensor representation (7) of decomposition (5). The proposed approach consists of iteratively fitting this 3rd order storage of the hexacovariance tensor using the Levenberg-Marquardt (LM) method[2].We consider the minimization of the following quadratic cost function:

$$f(\mathbf{p}) = \frac{1}{2}\|\mathbf{e}(\mathbf{p})\|_F^2 = \frac{1}{2}\mathbf{e}^{\mathrm{H}}(\mathbf{p})\mathbf{e}(\mathbf{p}), \tag{10}$$

where $\mathbf{e}(\mathbf{p}) = \mathrm{vec}\big(\widehat{\mathbf{C}}_{\mathbf{y}}^{'} - (\widehat{\mathbf{H}}^{(1)} \odot \widehat{\mathbf{H}}^{(2)})\widehat{\mathbf{H}}^{(3)\mathrm{T}}\big) \in \mathbb{C}^{M^6\times 1}$ is the residue and $\mathbf{p}$ is the parameter vector defined as:

$$\mathbf{p} = \begin{bmatrix} \mathbf{p}_{\widehat{\mathbf{H}}^{(1)}} \\ \mathbf{p}_{\widehat{\mathbf{H}}^{(2)}} \\ \mathbf{p}_{\widehat{\mathbf{H}}^{(3)}} \end{bmatrix} = \begin{bmatrix} \mathrm{vec}(\widehat{\mathbf{H}}^{(1)\mathrm{T}}) \\ \mathrm{vec}(\widehat{\mathbf{H}}^{(2)\mathrm{T}}) \\ \mathrm{vec}(\widehat{\mathbf{H}}^{(3)\mathrm{T}}) \end{bmatrix} \in \mathbb{C}^{3MP\times 1}, \tag{11}$$

and the LM update is given as follows:

$$\mathbf{p}(i+1) = \mathbf{p}(i) - \big[\mathbf{J}^H(i)\mathbf{J}(i) + \lambda(i)\mathbf{I}\big]^{-1}\mathbf{g}(i), \tag{12}$$

where $\mathbf{J}(i)$ denotes the Jacobian matrix:

$$\mathbf{J}(i) = \Big[\mathbf{J}_{\widehat{\mathbf{H}}^{(1)}}(i) \quad \mathbf{J}_{\widehat{\mathbf{H}}^{(2)}}(i) \quad \mathbf{J}_{\widehat{\mathbf{H}}^{(3)}}(i)\Big] \in \mathbb{C}^{M^6\times 3MP}, \tag{13}$$

and $\mathbf{g}(i)$ denotes the gradient vector:

$$\mathbf{g}(i) = \begin{bmatrix} \mathbf{g}_{\widehat{\mathbf{H}}^{(1)}}(i) \\ \mathbf{g}_{\widehat{\mathbf{H}}^{(2)}}(i) \\ \mathbf{g}_{\widehat{\mathbf{H}}^{(3)}}(i) \end{bmatrix} \in \mathbb{C}^{3MP\times 1} \tag{14}$$

After convergence, an estimate of the mixture $\widehat{\mathbf{H}}$ up to column permutation and scaling is obtained from the estimated parameter vector $\mathbf{p}_{\widehat{\mathbf{H}}^{(3)}}$.

[2]Note that the LM algorithm has been used in a different context to fit 3rd order tensors in [14].

Estimation error			
Data block length (K)	200	5000	20000
6-BIOME class.	0.0860	0.0469	0.0360
6-BIOME cyclo.	0.0729	0.0388	0.0226
PARAFAC-LM class.	0.0634	0.0392	0.0327
PARAFAC-LM cyclo.	0.0573	0.0279	0.0226

Table 1. Estimation error for 6-BIOME and PARAFAC-LM using classical and cyclostationary hexacovariance estimators.

BER		
(P, M)	$(3, 2)$	$(4, 3)$
6-BIOME class.	0.0653	0.1103
6-BIOME cyclo.	0.0503	0.0771
PARAFAC-LM class.	0.0315	0.0914
PARAFAC-LM cyclo.	0.0248	0.0676
Perfect MAP ($\mathbf{H}$ known)	0.0137	0.0064

Table 2. BER performance of 6-BIOME and PARAFAC-LM obtained with a MAP estimator. SNR=10dB.

5. SIMULATION RESULTS

We evaluate the performance of 6-BIOME and PARAFAC-LM algorithms when using the cyclostationarity-based hexacovariance estimator proposed in Section 3. The results were obtained from 100 Monte Carlo runs. For each run, the noisy mixture of cyclostationary sources is generated from a simulator of radiocommunication signals developed by A.Chevreuil [15], which allows the control of the transmission parameters as well as the parameters defining the radio channel. We have considered a reference carrier of 10MHz and a common symbol period of $T = 0.4$ ms. The sources are modulated using Binary Phase Shift Keying (BPSK). The pulse shaping filter is a square root raised cosine with roll-off factor 0.3 and the sampling rate of the observed data at the receiver is $T_s = T/2$. A uniform linear array of sensors separated by half a wavelength is considered. At each run, the angles of arrival of the sources are randomly drawn between 0° et 80° according to a uniform distribution. The Signal-to-Noise Ratio (SNR) is fixed at 10 dB in all simulations. At each run, the performance is evaluated from the following normalized error measure:

$$\mathrm{e}(\mathbf{H}, \widehat{\mathbf{H}}) = \frac{\|\mathbf{H} - \widehat{\mathbf{H}}\|_F^2}{\|\mathbf{H}\|_F^2}.$$

First, we are interested in evaluating the estimation error assuming 3 sources and 2 sensors. Table 1 shows the median value of the estimation error obtained with 6-BIOME and PARAFAC-LM algorithms using both classical ("class") and cyclostationary ("cyclo") hexacovariance estimators for different data block lengths. It can be seen that the exploitation of higher-order cyclostationarity improves the performance of both algorithms. Note also that the proposed PARAFAC-LM algorithm offers better results than 6-BIOME. Table 2 shows the Bit-Error-Rate (BER) performance of 6-BIOME and PARAFAC-LM algorithms averaged over 100 Monte Carlo runs. For each run, the BER is calculated *a posteriori* using a Maximum-a-Posteriori (MAP) sequence estimator based on the estimated mixing matrix using 5000 observations. As a reference for comparison, the performance of the perfect MAP estimator, which assumes perfect knowledge of the mixing matrix, is shown. For both algorithms, an improved BER performance is observed when higher-order cyclostationarity is exploited. Such a BER improvement is more pronounced in the case $(P, M) = (4, 3)$ where the performance of the classical hexacovariance estimator is limited due to the higher number of parameters to be estimated.

6. CONCLUSION

We have addressed the problem of blind identification of underdetermined mixtures in a cyclostationary context by exploiting sixth-order statistics. A "corrected" hexacovariance estimator has been presented which takes into account the second- and fourth-order cyclic moments. Using the proposed estimator, we have assessed the performance of 6-BIOME and PARAFAC-LM algorithms relying on different matrix factorizations of the estimated hexacovariance. Our results show that both algorithms benefit from the exploitation of higher-order cyclostationarity, thus offering an improved identification of the mixing matrix. We have also observed an improved performance of PARAFAC-LM over the 6-BIOME algorithm.

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